Math behind kinetic energy penetrators

The deceleration is relative to the square of the velocity

$$\frac{dv}{dt} = cv^2, \qquad c < 0$$

$$\frac{dv}{v^2} = c\,dt$$

Integrating both sides:

$$-\frac{1}{v(t)} = ct + a$$

$$v(t) = \frac{1}{-ct - a}$$

At t=0:

$$v(0) = v_0 = \frac{1}{-c \cdot 0 - a}$$

$$v_0^{-1} = -a$$

$$v(t) = \frac{1}{v_0^{-1} - ct}$$

Using the velocity function we can calculate the distance traveled from the starting point $s(0) = 0$:

$$s(t) = \int v(\tau) d\tau$$

$$s(t) = \int \frac{1}{v_0^{-1} - c\tau} d\tau$$

$$s(t) = \frac{1}{c} \ln \frac{1}{v_0^{-1} - ct} + s_0$$

At time t=0 the distance s should be equal to 0:

$$s(0) = \frac{1}{c} \ln \frac{1}{v_0^{-1}} + s_0 = 0$$

$$s_0 = -\frac{1}{c} \ln \frac{1}{v_0^{-1}}$$

$$s(t) = \frac{1}{c}\ln\frac{1}{v_0^{-1} - ct} - \frac{1}{c}\ln\frac{1}{v_0^{-1}}$$

$$s(t) = \frac{1}{c}\ln\frac{v_0^{-1}}{v_0^{-1} - ct}$$

$$s(t) = \frac{1}{c}\ln\frac{1}{1 - \frac{ct}{v_0^{-1}}}$$

$$s(t) = -\frac{1}{c}\ln(1 - ctv_0)$$

Then we can use these equations to calculate velocity at given distance

$$\begin{cases} s(t) = -\frac{1}{c}\ln(1 - ctv_0) \\ v(t) = \frac{1}{v_0^{-1} - ct} \end{cases}$$

$$v = \frac{1}{v_0^{-1} - ct}$$

$$v_0^{-1} - ct = \frac{1}{v}$$

$$ct = v_0^{-1} - \frac{1}{v}$$

$$t = \frac{1}{c}v_0^{-1} - \frac{1}{cv}$$

$$t = \frac{1}{cv_0} - \frac{1}{cv}$$

$$t = \frac{v - v_0}{cvv_0}$$

$$s(v) = -\frac{1}{c}\ln\left(1 - cv_0\frac{v - v_0}{cvv_0}\right)$$

$$s(v) = -\frac{1}{c}\ln\left(1 - \frac{v - v_0}{v}\right)$$

$$s(v) = -\frac{1}{c}\ln\left(\frac{v_0}{v}\right)$$

And hence

$$cs = -\ln\left(\frac{v_0}{v}\right)$$

$$cs = \ln\left(\frac{v}{v_0}\right)$$

$$e^{cs} = \frac{v}{v_0}$$

$$v = v_0 e^{cs}$$

Case example: Rheinmetall 120mm DM53 round shot from L/55 barrel has been given (in the Internet) deceleration of approx. 55 m/s @ 1km. Muzzle velocity is 1.75 km/s. What is the *c* value for that round?

$$c = \frac{1}{s}\ln\frac{v}{v_0}$$

$$c = \frac{1}{1}\ln\frac{1.75 - .055}{1.75} = -0.03193305$$

(unlikely low result, penetration would be over 80% of maximum @ 3km)

The penetration for large caliber APFSDS rounds has been approximated to depend on velocity by following formula: (http://ciar.org/ttk/mbt/papers/lakowski.2006-09/Penetration_Limits_of_Conventional_Large_Caliber_Anti_Tank_-_Kinetic_Energy_Projectiles.pdf)

$$P(v) \propto e^{\left(-\frac{1}{v^2}\right)}$$

That is, everything else being constant the penetration depends on velocity by above formula. This allows us to calculate the “absolute penetration” p_0 (or the limit for penetration as v goes toward infinity) and then produce the penetration value by multiplication for every velocity given by formulas produced earlier.

$$p_0 = \frac{P(v_0)}{e^{\left(-\frac{1}{v_0^2}\right)}}$$

Where $P(v_0)$ is the penetration at zero distance and at muzzle velocity v_0. For other velocities we then use the p_0 to calculate the penetration at velocity:

$$P(v) = p_0 e^{\left(-\frac{1}{v^2}\right)}$$

And by aforementioned formulae

$$v = e^{cs} \Rightarrow$$

$$P(s) = p_0 e^{\left(-\frac{1}{(e^{cs})^2}\right)} = p_0 e^{\left(-\frac{1}{e^{2cs}}\right)}$$

If we have initial velocity v_0, drag constant c and any penetration value for any range we can calculate the distance/penetration table for that shot. If either v_0 or c is missing we need two reference points and if we only have penetration values the calculation is not solvable in closed form and the performance will be poor as the deviations of measurements will severely affect the results.

See also: http://www.proteusphototech.com/wp-content/uploads/2013/06/MIL-STD-662F.pdf (ammunition speed at various measurement points)

Problems: The penetration equations work only for APFSDS type of ammunition L/D must be between 10 and 20 and speed above Mach 1.1: regular AP shots may or may not work out well.